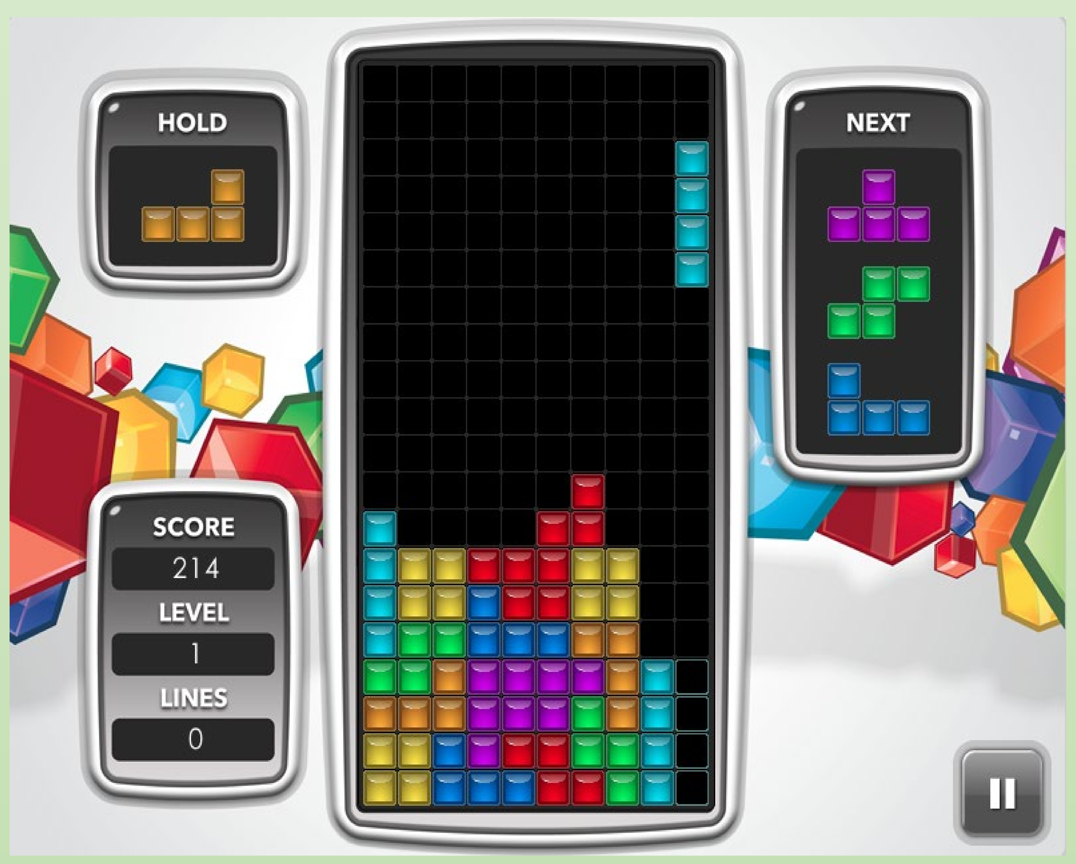
The Difficulty of Winning Tetris

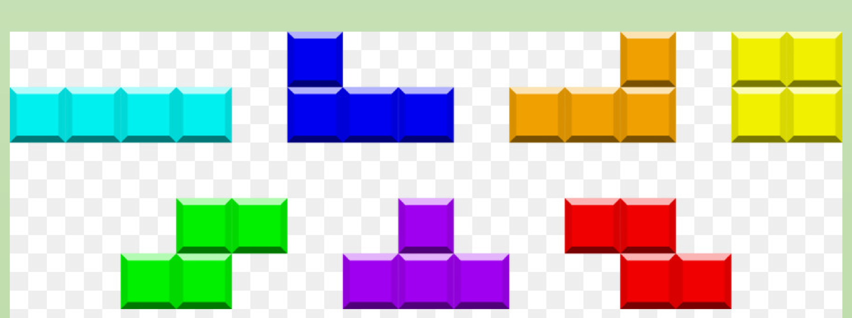
John Michael Visperas and advisor Russell May
Morehead State University
Department of Mathematics and Physics

Description of game

Tetris is a classic game that uses seven unique blocks that can be arranged in a game board. The pieces fall from above and the player arranges them in a way that will fill the rows. Each completely filled row generates points according to its level and then disappears. The tiles above take its place. After a player clears ten rows, a new level is started. As the levels increase, the Tetris pieces fall faster.

The goal of our modified Tetris game is to score higher than the average score with regards to the probability of losing the game.





Important Theorems

"There is no winning strategy in Tetris if the computer is aware and reacting to the player's actions."

– Brzustowski

"Given a game consisting of p pieces, approximating the maximum number of pieces that can be placed without a loss of $\mathbf{p^{1-\varepsilon}}$ for any constant $\boldsymbol{\varepsilon > 0}$ is NP- Hard".

–Demaine

Tetris theorems work for the most generalized Tetris game, even for newer versions of the game.
– Burgiel

Modified Tetris Game

In our modified version of Tetris the points a player gets for clearing a level correspond to the level itself. For example, a player gets five points for clearing the fifth level. Other scoring mechanics are not calculated in this version. On each of the levels, the player has a fixed chance *p* of winning that level.

Summation Model

The summation formula for the average score is modeled after a 2nd derivative a geometric series. The probability of losing the game on the *nth* level is

$\boldsymbol{q * p^{n-1}}$, and the score would be a total of

$$\mathbf{1 + 2 + \cdots + n - 1 = \left(\frac{n-1}{2}\right) * n}$$

points.

Average Score Summation

The average score is found by adding the probability of concluding the game at the *nth* level times the total score for the previous levels:

$$\sum_{n=1}^{\infty} q * p^{n-1} * \left(\frac{n-1}{2}\right) * n$$

This sum is also related to the second derivative of a geometric series.

$$\sum_{n=0}^{\infty} p^n = \frac{1}{1-p}$$

So the average score is:

$$\frac{p}{q^2}$$

This summation formula gives an average score for the game that depends on the difficulty level *p*.

How to Win

In this specified Tetris game, as long as the player can exceed the average game score, then they have won at Tetris.

H	I	J	K	L
Average Score Summation				
0.2	0.2 is Probability of First Loss			
Game Level	Probability of losing on the current Level	Score at the current level	Probability x Score	Score Multiplier
1	0.2	0	0	800
2	0.16	800	128	
3	0.128	2400	307.2	
4	0.1024	4800	491.52	
5	0.08192	8000	655.36	
6	0.065536	12000	786.432	
7	0.0524288	16800	880.80384	
8	0.04194304	22400	939.524096	
9	0.033554432	28800	966.3676416	
10	0.026843546	36000	966.3676416	
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121	4.69709E-13	5808000	2.72807E-06	
		Calculated Score	15999.99999	
		Theoretical Score	16000	

References

Burgiel, Heidi.
"How to Lose at Tetris."

Demaine, Erik D., et. al.
"Tetris Is Hard, Even to Approximate."

Brzustowski, John.
"Can You Win at Tetris?"